Problem Set 3 - LV 141.A55 QISS - 4.4.2016

1. **Lindblad Equation**

   Consider a qubit

   $$H_0 = \frac{\hbar\omega_0}{2}\left(|1\rangle\langle 1| - |0\rangle\langle 0|\right)$$

   with a classical driving field

   $$H_\mathrm{d} = \frac{\hbar\omega_1}{2}\left(|1\rangle\langle 0|e^{i\omega t} + |0\rangle\langle 1|e^{-i\omega t}\right)$$

   Furthermore consider relaxation with the Lindblad operator

   $$L_r = \sqrt{\gamma_r}|0\rangle\langle 1|$$

   and dephasing

   $$L_\phi = \sqrt{\gamma_\phi}(|0\rangle\langle 0| - |1\rangle\langle 1|)$$

   Write the Lindblad equation in the basis

   $$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

   Show that you can write the four coupled ordinary differential equations in the form

   $$\frac{\partial}{\partial t}\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} = -i\omega_0\mathbf{U_0}\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} - i\omega_d\mathrm{U}_\mathrm{d}\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} + \gamma_r\mathbf{L}_r\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} + \gamma_p\mathbf{L}_p\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix}$$

2. **Qubit Simulation - Relaxation**

   Use Python to solve the ordinary differential equation (`scipy.integrate.ode`).

   Now solve the time evolution of the qubit state without driving starting in the state

   $$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

   Use the following parameters to simulate relaxation

   ```
   gammap=0;gammar=0.1;omega0=1;omega1=0;omega=0;
   ```

   Plot $\langle s_z\rangle$, $\mathrm{tr}(\rho)$ and $\mathrm{tr}(\rho^2)$ versus time.

3. **Qubit Simulation - Dephasing**

   Now we add dephasing to the qubit

   ```
   gammap=0.1; gammar=0.1; omega0=1; omega1=0; omega=0;
   ```

Plot $\langle s_x \rangle$, $\langle s_y \rangle$ and $\sqrt{\langle s_x \rangle^2 + \langle s_y \rangle^2}$ versus time. Comment your result.

4. **Qubit Simulation - Rabi oscillation**

   Finally we add a oscillatory driving field causing Rabi cycles. Start in the ground state, i.e. $|0\rangle$

   Plot $\langle s_z \rangle$ for a strong driving

   ```
   gammap=0.1; gammar=0.1; omega0=1; omega1=2; omega=1;
   ```

   and weak driving

   ```
   gammap=0.1; gammar=0.1; omega0=1; omega1=0.5; omega=1;
   ```